Conservation of Energy Part 1

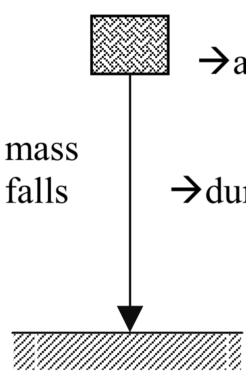
In previous grades, you learned the famous statement "energy is neither created nor destroyed, only transferred from one form to another". This really means that for any given event, the total energy contained in a system is constant, regardless of how different types of energy change during the event. This is similar to the conservation of momentum theorem that was dealt with in the last section, except that since energy is a *scalar* quantity, no vector diagrams are required!

However, unlike momentum (which has only one form and one equation), there are many forms of energy in nature. In Physics 12 however, we are given only two equations for two types of energy: $\mathbf{E_p}$ and $\mathbf{E_k}$. This limits our ability to utilize the conservation of energy theorem.

Essentially, we can only examine systems or events where gravitational and kinetic energies are involved, as well as the heat energy produced when friction occurs. Such systems include: falling objects, roller coasters/ramps, pendulums and slides.

To simplify things, in this section we will only examine conservation of energy problems in situations where friction can be ignored.

Consider an object dropped from a height '**h**' above the ground. When this mass falls from rest and loses vertical height, the loss of gravitational potential energy ($\mathbf{E_p}$) is converted entirely to kinetic energy ($\mathbf{E_k}$).



→at rest, $\mathbf{E_k = 0} \;\therefore\; \mathbf{E_{total} = E_p = mgh}$

→during the fall, $\mathbf{E_p}$ is converted to $\mathbf{E_k}$

→at impact, $\mathbf{E_p = 0} \quad \therefore \quad \mathbf{E_{total} = E_k = \frac{1}{2} mv_f^2}$

- Since total energy remains the same from start to finish, in this example:

$$\mathbf{mgh = \frac{1}{2} mv_f^2}$$

Another way of looking at conservation of energy is to consider how energy is gained or lost. In the above example:

$$\Delta \mathbf{E_p} \text{ lost} = \Delta \mathbf{E_k} \text{ gained} \quad \rightarrow \quad \mathbf{mg\Delta h = \frac{1}{2} mv_f^2}$$

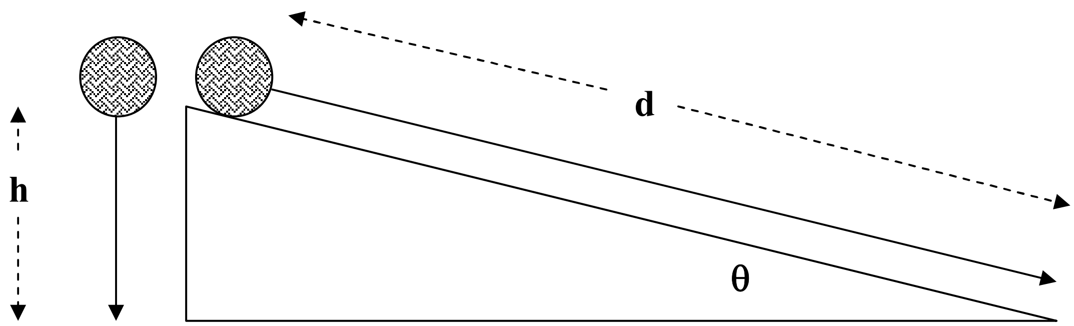
→Note that there is no initial speed $\mathbf{v_i}$ in this situation.

Example #11: A 35 kg mass falls 4.0 m to the ground.

a) **How much kinetic energy does it have when it strikes the ground?**
b) **With what speed does it strike the ground?**

(see Work-Energy Ex 11 for answer)

Now consider two identical balls: one dropped from a height ‘**h**’, the other rolling from rest at the same height down a frictionless incline of length ‘**d**’.



It can be proven algebraically that with no friction, the final speed of each ball will be the same!

- Using kinematics, $\mathbf{v_f^2 = v_i^2 + 2ad}$ → where $\mathbf{v_i = 0}$
- Therefore, $\mathbf{v_f = \sqrt{2ad}}$
- For the left ball, $\mathbf{a = g}$ and $\mathbf{h = d\sin\theta}$ so $\mathbf{v_f = \sqrt{2gd\sin\theta}}$
- For the right ball, $\mathbf{a = g\sin\theta}$ so once again, $\mathbf{v_f = \sqrt{2gd\sin\theta}}$

This means that, so long as there is no friction, the speed of an object travelling on any path depends only on its change in height. Conservation of energy can be used to solve for unknown values based on this knowledge.

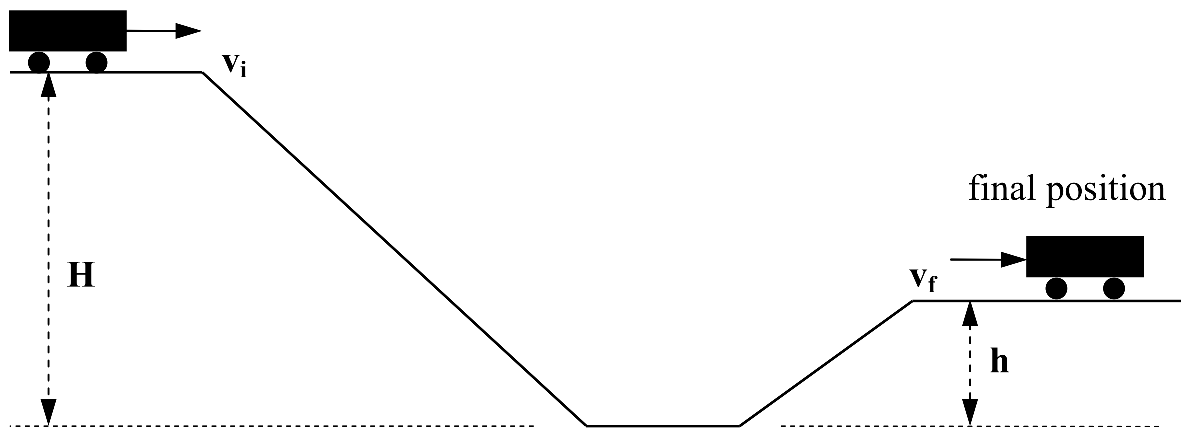
Keep in mind though, these two points:

- We are only referring to speed, and not velocity, which is a vector quantity.
- If there *is* significant friction, this shortcut for finding speed will not work.

Example #12: In the diagram above, if the right ball has a mass of 5.2 kg and an initial speed of 1.4 m/s at the top of the 2.8-m high ramp, what will its speed be at the bottom of the ramp?

(see Work-Energy Ex 12 for answer)

Here's a slightly more complex problem: energy conservation on a roller coaster. Once again, assume friction is negligible, as well as wind resistance, etc.



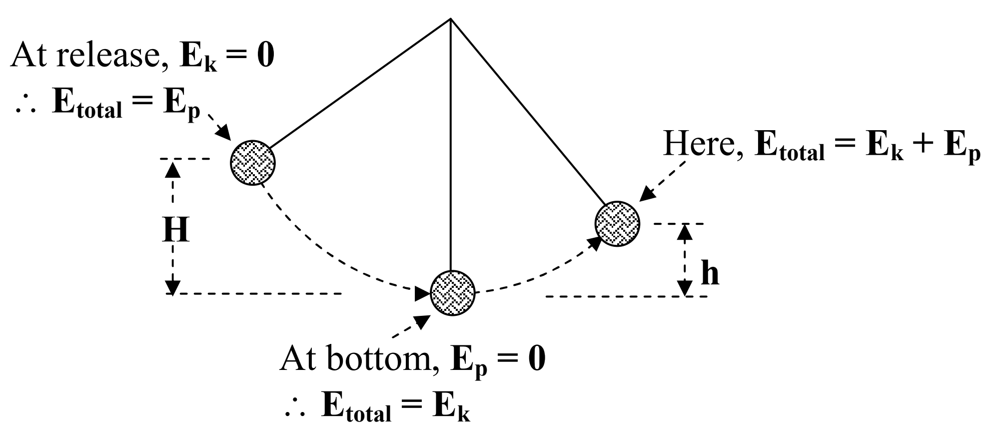
Since the total energy is constant, and since friction is negligible,

the sum of $\mathbf{E_p} + \mathbf{E_k}$ before = sum of $\mathbf{E_p} + \mathbf{E_k}$ after

Example #13: If a cart of mass 10 kg and with an initial speed of 3.5 m/s rolls down a 50 m high frictionless incline and then proceeds to roll up another similar incline to a height of 20 m, what is the speed of the cart at this point?

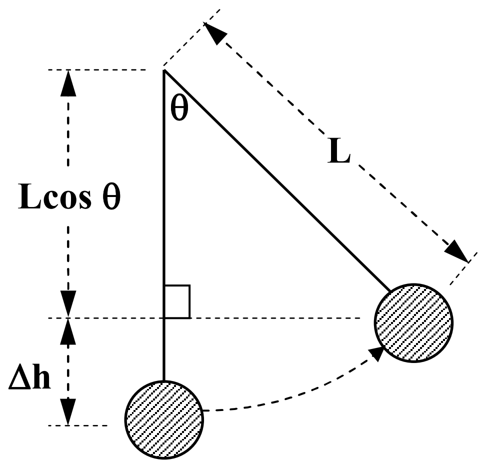
(see Work-Energy Ex 13 for answer)

Finally, we'll look at energy conservation in a frictionless pendulum.



Note the following:

- Height is measured from the bottom of the pendulum's swing.
- At any point, the sum of $\mathbf{E_k + E_p}$ = **total energy**.
- Since **total energy** is constant (cons. of energy):
  $\mathbf{E_p}$ at highest point = $\mathbf{E_k}$ at lowest point
- The change in height Δ**h** of a pendulum can be determined if the pendulum's length '**L**' is known as well as the angle θ (from vertical) to which it was raised.



$$\Delta h = L - L \cos \theta$$

$$\Delta h = L(1 - \cos \theta)$$

Example #14: A pendulum bob of mass 5.0 kg falls through a height of 25 cm as it swings from maximum height to lowest position.

a) How fast is it going at the bottom?
b) What is the energy of the bob at the bottom of the swing?
c) What is the speed of the bob as it swings up past the bottom of its arc and rises 10 cm from the bottom position?
d) What is the total energy at this position?
e) What is the potential energy at this position?

(see Work-Energy Ex 14 for answer)

Finally, be clear on this: these frictionless systems do not exist, except at the sub-atomic level. If they did, they would be described as *perpetual motion* systems that would continue to move without any additional energy required.